QMW-PH-97-8
hep-th/9703004
February 1997

Deformations of a 2D charged black hole

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Abstract

We discuss two types of deformations of a 2D black hole carrying an electric charge. Type I gives rise to a space-time configuration similar to the one described by McGuigan, Nappi and Yost. Whereas type II results in a space-time configuration which has a rather peculiar geometry.

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1 Introduction

In string theory, in order to describe D-dimensional space-time configurations with a number of various charges, one has to consider a target space with extra N dimensions which accommodate corresponding internal degrees of freedom. Therefore, if, for example, one wants to study a 2D black hole carrying, say one electric charge, one has to take a string propagating in a three (or more) dimensional target space with the topology $R^2 \times S^1$, where S^1 is a circle embedding the electric charge. Extra dimensions give rise to new properties of black holes. Namely, by virtue of extra dimensions, stringy black holes are allowed to have hair [1],[2]. This fact means that a given 2D black hole string solution admits non-trivial perturbations along extra dimensions. It is argued that these dimensions can be responsible for entropy of the black hole as well as they can be the place where the information is stored [1],[2],[3].

Apparently, some of these deformations can be studied by means of conformal perturbation theory. Indeed, form the point of view of the sigma model approach, various deformations of string solutions are nothing but perturbations of given conformal sigma models. There are truly marginal and relevant perturbations which preserve the consistency of the non-linear sigma model as a two dimensional quantum field theory.

All truly marginal perturbations of a given conformal field theory, by definition, form the moduli space of string solutions. Changing coordinates in this space does not change the given conformal sigma model. At the same time, relevant perturbations change the CFT simply by breaking the conformal symmetry. However, there may be a situation when a non-conformal field theory flows to another critical point in the infrared limit. This IR CFT can be again a certain string solution. Thus relevant perturbations can take a string solution of one type to another string solution of a different type. In this case, by studying all possible relevant perturbations on a given string configuration, one can learn about dynamical properties of string theory.

The Witten's 2D black hole is an interesting example of a non-trivial string solution [4]. Therefore, it might be instructive to look at the effect of relevant perturbations on this 2D black hole.

In the present paper, we would like to discuss two types of relevant perturbations of the Witten's 2D black hole. As we shall see both these perturbations have one thing in common - electric field. However, their effects on the 2D black hole will be dramatically distinct.

2 The basic conformal action

In string theory a 2D black hole without electric charge can be described as an $SL(2)/U(1)$ coset [4], which in turn is formulated as a gauged Wess-Zumino-Novikov-Witten model [5],[6]. The action of the gauged WZNW model is given as follows

$$S(g,A) = S_{WZNW} \ + \ \frac{k}{2\pi}\int d^2z \mathrm{Tr}\left[Ag^{-1}\bar{\partial}g - \bar{A}\partial g g^{-1} + Ag^{-1}\bar{A}g - A\bar{A}\right], \tag{2.1}$$

where

$$S_{WZNW} = \frac{k}{8\pi}\int d^2z \mathrm{Tr} g^{-1}\partial^\mu g g^{-1}\partial_\mu g \ + \ \frac{ik}{12\pi}\int d^3z \mathrm{Tr} g^{-1}dg\wedge g^{-1}dg\wedge g^{-1}dg \tag{2.2}$$

and $g\in SL(2)$, A, $\bar{A}$ are the gauge fields taking values in the $U(1)$ algebra (compact or noncompact).

In order to be able to describe a 2D black hole carrying an electric charge, we have to extend the dimensionality of the target space to three. This can be done by adding to the action (2.1) a free scalar compactified on a unit circle. Then, the action of an $e=0$ (which means that there is no yet an electric field) 2D black hole will be given as follows

$$S_{3D} = S(g,A) \ + \ S_{S^1}, \tag{2.3}$$

where

$$S_{S^1} = \frac{i}{4\pi}\int d^2z\bar{\partial}y\partial y. \tag{2.4}$$

The CFT described by eq. (2.3) is characterized by the Virasoro central charge

$$c = \frac{3k}{k+2}. \tag{2.5}$$

Formally, the given central charge coincides with the central charge of the ordinary WZNW model on $SL(2)$ at level k. As one can immediately see formula (2.5) differs from the

central charge of the Witten's black hole

$$c_W = \frac{3k}{k+2} \; - \; 1. \tag{2.6}$$

In particular, this difference results in the new value of k in the critical case

$$\frac{3k}{k+2} = 26 \quad \longrightarrow \quad k = -\frac{52}{23} = -2.26..., \tag{2.7}$$

whereas for the Witten's black hole

$$k_W = -\frac{9}{4} = -2.25. \tag{2.8}$$

3 $e \neq 0$

The CFT given by eq. (2.3) describes a 2D black hole without any electric excitations. This will be our basic conformal model. When the electric excitations are turned on the sigma model action acquires a new term corresponding to the configuration of the electric field. Generically, this term has the following structure

$$S(e) = S_{3D} \; - \; e\int d^2z \; B_\mu(x)\partial x^\mu \bar{J}_c, \qquad \bar{J}_c = i\bar{\partial}y, \tag{3.9}$$

and B_μ is the vector gauge potential, a function of x^μ only, with x^μ being the coordinates of the 2D target space, and e is proportional to the electric charge. Admissible configurations of B_μ are fixed by the conformal invariance of the non-linear sigma model in eq. (3.9).

If we choose e to be a small parameter, then one can treat model (3.9) within perturbative approach. In this case, one has to consider all possible relevant deformations of the form (3.9) constructed in terms of the CFT S_{3D}. A natural choice appears to be as follows

$$B_\mu \partial x^\mu \bar{J}_c \equiv O_B(z,\bar{z}) = L_{ab} J^a(z) \phi^{b\bar{3}}(z,\bar{z}) \bar{J}_c, \tag{3.10}$$

where

$$J = -\frac{k}{2}\partial g g^{-1}, \qquad \phi^{a\bar{a}} = \mathrm{Tr}(t^a g t^{\bar{a}} g^{-1}), \tag{3.11}$$

and

$$L = \frac{1}{A}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{4}{k} \end{pmatrix}. \tag{3.12}$$

where A is a normalization constant. The form of the matrix L is fixed by the invariance of the operator O_B under the gauge group $U(1)$.

The important point to be made is that the conformal dimension of the operator O_B is

$$\Delta = 1 \ + \ \frac{2}{k+2}, \tag{3.13}$$

that is for all $k < -2$, O_B is a relevant operator. Moreover, for all $k < -4$, this is a relevant operator with positive conformal dimension. In what follows, we shall be interested in the limit $k \to -\infty$. In this case, O_B is a relevant quasimarginal operator. From the point of view of the string criticality condition, this limit means that we consider an electrically charged 2D black hole with 24 additional internal dimensions one of which accommodates the electric charge, whereas the other 23 charges are set to zero. In total, the dimensionality of the whole target space is equal to 26. Thus, we are dealing with a certain solution of the critical bosonic string.

Now we are going to study possible relevant perturbations of type (3.9). It turns out that perturbation theory allows two different types of deformations of the space-time configuration of the charged 2D black hole(s).

3.1 $e \neq 0$ *type I*

By adding the relevant operator O_B to the CFT S_{3D} we do not automatically obtain a renormalizable quantum field theory. Indeed, the operator O_B has the following operator product expansion (see appendix)

$$O_B(z,\bar{z})O_B(0) = \frac{A^2}{2(1+k/2)|z|^{2\Delta}}O(0) \ + \ \mathrm{I} \ + \ ..., \tag{3.14}$$

where

$$O(z,\bar{z}) = L_{ab}L_{\bar{a}\bar{b}}J^a\bar{J}^{\bar{a}}\phi^{a\bar{a}} \tag{3.15}$$

is a new operator with the same conformal dimension as in eq. (3.13), I is the unity operator and dots stand for all other operators with conformal dimensions greater than 1. Thus, the CFT S_{3D} perturbed by O_B alone is not renormalizable and one has to add also the operator O with the corresponding coupling constant. From the physical point

of view, this operator corresponds to the back reaction of the 2D metric to the electric charge. Then, the perturbed theory takes the following form

$$S_I(e,\epsilon) = S_{3D} \; - \; \epsilon \int d^2z O(z,\bar{z}) \; - \; e \int d^2z O_B(z,\bar{z}). \tag{3.16}$$

The given QFT is renormalizable by virtue of the fact that the operators O, O_B form a closed algebra with respect to their OPE's:

$$\begin{aligned} O(z,\bar{z})O(0) &= \frac{1}{|z|^{2\Delta}} O(0) \; + \; \mathrm{I} \; + \; ..., \\ O(z,\bar{z})O_B(0) &= \frac{1}{|z|^{2\Delta}} O_B(0) \; + \; ... \end{aligned} \tag{3.17}$$

We shall consider the theory in eq. (3.16) as type I. Its characteristic features are that the perturbation (3.16) does not excite the metric component of the hidden dimension, however, it generates a non-trivial WZ term. Indeed, the operator O_B is not symmetric in ∂ and $\bar{\partial}$.

Given the OPE's (3.14), (3.17), it is not difficult to calculate the corresponding renormalization group beta functions of e and ϵ. We find

$$\begin{aligned} \beta_\epsilon &= (2-2\Delta)\epsilon \; - \; \pi\epsilon^2 \; - \; \frac{\pi A^2}{k} e^2 \; + \; ..., \\ \beta_e &= (2-2\Delta)e \; - \; \pi\epsilon e \; + \; ... \end{aligned} \tag{3.18}$$

If we assume that e is very small, then one can ignore the e^2 contribution to β_ϵ in the limit $k \to -\infty$. In this limit, one can easily see that the equations (3.18) admit a non-trivial infrared conformal point at

$$\epsilon^\star = \frac{2-2\Delta}{\pi} \; + \; ... = -\frac{4}{\pi k} \; + \; \mathcal{O}(1/k^2). \tag{3.19}$$

Note that when ϵ is set to its critical value $\epsilon^\star$, the charge e gets no restriction on its value apart from the requirement to be small. In other words, at the point $\epsilon^\star$, the perturbed theory(3.16) becomes a CFT with a continuous parameter e. Correspondingly, the Virasoro central charge at the IR critical point is given as follows (see appendix)

$$c(\epsilon^\star) = \frac{3k}{k+2} \; + \; \frac{256}{3k} \; + \; \mathcal{O}(1/k^2). \tag{3.20}$$

Unfortunately, the given perturbative value of the Virasoro central charge does not appear to fit any known exact CFT. However, from the string theory point of view, at the IR fixed point, the perturbed CFT (3.16) describes a 2D electrically charged black hole whose metric looks similarly to the metric found in [7].

3.2 $e \neq 0$ *type II*

There exists another deformation of the CFT S_{3D} which differs from the perturbation described above. Let us consider the following operator

$$O_+(z,\bar{z}) = \Big(\frac{1}{2}L_{ab}J^a(z)\phi^b(z) \;+\; x\phi^3(z)J_c(z)\Big)\overline{\Big(\frac{1}{2}L_{ab}J^a(z)\phi^b(z) \;+\; x\phi^3(z)J_c(z)\Big)}, \tag{3.21}$$

where the parameter x is defined from the condition

$$O_+(z,\bar{z})O_+(0) = \frac{1}{|z|^{2\Delta}}O_+(0) \;+\; \mathrm{I} \;+\; ... \tag{3.22}$$

This condition is satisfied when

$$x = \frac{\pm i\sqrt{|k|}}{2A}. \tag{3.23}$$

The operator O_+ has the conformal dimension as in eq. (3.13).

Now we can consider the following deformation

$$S_{II}(\mu) = S_{3D} \;-\; \mu\int d^2z O_+(z,\bar{z}), \tag{3.24}$$

where μ is a new coupling constant. It is not difficult to see that this perturbed CFT includes type I perturbation with

$$e = \mu x/2. \tag{3.25}$$

However, compared to the model in eq. (3.16), the theory (3.24) introduces another electric field $\bar{B}_\mu$ associated with the string modes of the opposite chirality in the compact dimension. Additionally, the operator O_+ gives rise to the excitation of the metric of the hidden dimension. Thus, the new perturbed CFT describes a few more processes occurring in the compact dimension. This can be viewed as a further deformation of the type I perturbation by extra operators

$$O_{\bar{B}} \equiv \bar{B}_\mu J_c\bar{\partial}x^\mu = L_{\bar{a}\bar{b}}\bar{J}^{\bar{a}}\phi^{3\bar{b}}J_c, \qquad O_{3\bar{3}} = \phi^{3\bar{3}}J_c\bar{J}_c. \tag{3.26}$$

Because O_+ forms a closed OPE algebra, the perturbed CFT (3.24) is a renormalizable QFT and one can compute the corresponding beta function. We find

$$\beta_\mu = (2 - 2\Delta)\mu \ - \ \pi\mu^2 \ + \ ... \tag{3.27}$$

This equation also exhibits a non-trivial IR fixed point at

$$\mu^\star = -\frac{4}{\pi k} \ + \ \mathcal{O}(1/k^2). \tag{3.28}$$

Interestingly, at this value of μ, the electric charge e is fixed at the value corresponding to the so-called extremal 2D black hole. Namely,

$$e = \frac{\pm 4i}{\pi A\sqrt{|k|}}. \tag{3.29}$$

At the IR conformal point, the most remarkable thing happens to the Virasoro central charge. At this point, it is given by the following perturbative formula (see appendix)

$$c(\mu^\star) = \frac{3k}{k+2} \ + \ \frac{12}{k} \ + \ \mathcal{O}(1/k^2). \tag{3.30}$$

This expression needs to be carefully examined before we can identify the IR conformal point with any exact CFT. There are two candidates which fit the perturbative expansion (3.30). Namely, the $SU(2)_{|k|}$ WZNW model and the $(SU(2)_{|k|}/U(1)) \times U(1)$ coset construction. Both the WZNW model and the coset construction have one and the same Virasoro central charge, which in the large $|k|$ limit has the form given by eq. (3.30). In order for the IR CFT to be the $SU(2)$ WZNW model, it has to have the corresponding affine currents. However, the perturbed theory does not appear to have affine currents which form the affine algebra $\widehat{SU(2)}$. At the same time, the perturbed theory (3.24) still has the BRST symmetry, since the perturbation operator is BRST invariant. Thus, at the IR conformal point one has to expect to arrive at a certain gauge invariant model whose gauge group has to be $U(1)$. Apparently, the $(SU(2)_{|k|}/U(1)) \times U(1)$ coset has all these features, provided that we have gauged away the compact subgroup of $SL(2)$ at the UV fixed point. Hence, we come to conclusion that at the IR critical point, the perturbed CFT coincides with $(SU(2)_{|k|}/U(1)) \times U(1)$ in the large $|k|$ limit. All in all, we can have the following picture of the corresponding renormalization group flow

$$\frac{SL(2)_k}{U(1)} \times U(1) \longrightarrow \frac{SU(2)_{|k|}}{U(1)} \times U(1). \tag{3.31}$$

The given flow describes the change the target space geometry undergoes under the perturbation by the operator O_+. Indeed, the UV conformal point of the flow corresponds to the Witten's black hole without electric charge, whose Euclidean target space geometry is presented as a cigar. After adding the perturbation of type II, we observe that along this flow the 2D black hole settles down to the configuration described by the coset $(SU(2)_{|k|}/U(1)) \times U(1)$, in the limit $k \to -\infty$. Naively, it may seem that the given coset describes a charged two dimensional sphear. However, Bardakci, Crescimanno and Rabinovici have shown [8] (see also [9]) that $SU(2)/U(1) \neq S^2$. In fact this coset has a rather peculiar geometry of a 2D sphear with blown up equator (or two Mexican hats with infinitely large brim glued together at the rim). Also it describes parafermions [10]. Unfortunately, not very much is known about string solutions with this type target-space geometry.

4 Conclusion

We have studied two types of perturbations of the Witten's 2D black hole. These perturbations have one thing in common - they excite an electric field. However, the difference between these two perturbations is in their fate as they approach the IR limit. Type I perturbation evolves into a configuration, which has an interpretation of the 2D charged black hole. Whereas type II perturbation, at the IR conformal point ceases to look like a 2D black hole, but acquires a rather peculiar target-space geometry.

We also would like to point out that we have not discussed how the considered perturbations affect the dilaton field. This may be important in order to understand whether these two perturbations are related to each other in any way.

5 Acknowledgments

I am indebted to Ian Kogan for very significant discussions.

Appendix

In this appendix we would like to explain how OPE's (3.14), (3.17) come into being. First of all, let us take the holomorphic operator

$$O(z) \equiv L_{ab} J^a(z) \phi^b(z), \tag{A.1}$$

where $\phi^a(z)$ is the holomorphic part of $\phi^{a\bar{a}}(z,\bar{z})$. One can demand that $O(z)$ obeys the following OPE

$$O(z)O(0) = \frac{1}{z^\Delta} O(0) \ + \ \mathrm{I} \ + \ ... \tag{A.2}$$

This requirement results in a condition on the matrix L_{ab} [11]

$$\begin{aligned} 2\left(\frac{k}{2} g^{k(m}\delta_p^{n)} \ - \ f_f^{k(m} f_p^{n)f}\right) L_{mn} = L_{ab} L_{cd} \Bigg\{ \frac{k}{2} \Big[g^{ka} C_p^{cd,b} \ + \ g^{kb} C_p^{cd,a} \\ + \ g^{kc} C_p^{ab,d} \ + \ g^{kd} C_p^{ab,a} \Big] \ - \ f_f^{k(a} f_e^{b)f} C_p^{cd,e} \ - \ f_f^{k(c} f_e^{d)f} C_p^{ab,c} \ + \ f_e^{k(a} C_p^{b)e,cd} \Bigg\}, \end{aligned} \tag{A.3}$$

with

$$\begin{aligned} C_d^{ab,c} \ &= \ \frac{A}{2}\left(f_e^{bc} f_d^{ea} \ + \ f_e^{ac} f_d^{eb}\right), \\ C_e^{ab,cd} \ &= \ \frac{A}{4}\Big[\left(f_e^{af} f_n^{bd} f_f^{cn} \ + \ f_e^{af} f_n^{bc} f_f^{dn} \ + \ f_e^{cf} f_n^{ad} f_f^{bn} \ + \ f_e^{df} f_n^{ac} f_f^{bn}\right) \ + \ (a \leftrightarrow b)\Big]. \end{aligned} \tag{A.4}$$

The matrix

$$L = \frac{1}{A}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{4}{k} \end{pmatrix} \tag{A.5}$$

is one of the solutions of eq. (A.3). The constant A can be computed from the normalization condition which stems from the classical identity

$$J^a = \phi^{a\bar{a}} \phi^{b\bar{a}} J^b. \tag{A.6}$$

This means that in the large k limit

$$\phi^{a\bar{a}} \phi^{b\bar{a}} = g^{ab}. \tag{A.7}$$

Finally, we find

$$A^2 = 1/c_V, \qquad c_V g^{ab} = -f_d^{ac} f_c^{bd}. \tag{A.8}$$

In the case under consideration, $c_V = 2$.

Alternatively, the matrix L_{ab} can be derived from the BRST condition

$$QO(0)|0\rangle = 0, \tag{A.9}$$

where the BRST charge Q is defined as follows

$$Q = \oint \frac{dw}{2\pi i} : c(\tilde{J}^3 \ + \ J^3) : (z). \tag{A.10}$$

Here c is the $U(1)$ ghost and $\tilde{J}^3$ is the current associated with the subgroup $H = U(1)$,

$$\tilde{J}^3(z)\tilde{J}^3(0) = \frac{-k/2}{w^2} \ + \ \text{reg.terms}. \tag{A.11}$$

Since ϕ^3 is also a BRST invariant operator, all OPE's involving O and ϕ^3 must be BRST invariant. This symmetry principle allows us to determine which operators may appear on the right hand side of OPE's. Then the exact OPE coefficients can be calculated from the consistency conditions [11]. So we find

$$\phi^3(z)\phi^3(0) = \frac{A^2}{(1+k/2)z^{\Delta_\phi - 1}}O(0) \ + \ \mathrm{I} \ + \ ... \tag{A.12}$$

Now it is easy to see that

$$\phi^{\bar{3}}(\bar{z})\bar{J}_c(\bar{z})\phi^{\bar{3}}(0)\bar{J}_c(0) \bar{=} \frac{A^2}{(2+k)\bar{z}^{\Delta}}\bar{O}(0) \ + \ \mathrm{I} \ + \ ... \tag{A.13}$$

Together with (A.2) this gives rise to the OPE in eq. (3.14).

Similarly, the OPE of O with O_B follows from the formula

$$\phi^c(z)O(0) = \frac{L_{ab}C_d^{ab,c}}{2z^{\Delta_\phi}}\phi^d(0) \ + \ ..., \tag{A.14}$$

where $C_d^{ab,c}$ is given by (A.4).

Let us also show how we compute perturbative Virasoro charges (3.20), (3.30). According to Zamolodchikov's c-theorem [13], the Virasoro central charge at the IR fixed point is given by the following formula [14]

$$c(IR) = c(UV) \ - \ \frac{(2-2\Delta_O)^3||O||^2}{C^3} \ + \ ..., \tag{A.15}$$

where $||O||$ is the norm of the perturbation operator and C is the coefficient in the following OPE

$$O(z,\bar{z})O(0) = \frac{C}{|z|^{2\Delta_O}}O(0) \ + \ .... \tag{A.16}$$

In type I perturbation, in the limit $k \to -\infty$, only the operator O contributes into leading order corrections at the IR conformal point. We find

$$C_I = 1, \qquad ||O||^2 = \frac{k^2 \sum_{i,\bar{i}=1}^{2} ||\phi^{i\bar{i}}||^2}{4A^4} \;+\; .... \tag{A.17}$$

Here the norm of the operator ϕ can be derived from the normalization condition (A.7) [15],

$$||\phi^{a\bar{a}}\phi^{b\bar{b}}|| = \frac{\delta^{ab}\delta^{\bar{a}\bar{b}}}{\dim G} \;+\; \mathcal{O}(1/k). \tag{A.18}$$

Thus,

$$c(IR)_I = c(UV) \;+\; \frac{256}{3k} \;+\; \mathcal{O}(1/k^2). \tag{A.19}$$

For type II we find

$$C_{II} = 1, \qquad ||O_+||^2 = \frac{k^2 \sum_{a\bar{a}=1}^{3} ||\phi^{a\bar{a}}||^2}{4^3 A^4} \;+\; .... \tag{A.20}$$

Correspondingly,

$$c(IR)_{II} = c(UV) \;+\; \frac{12}{k} \;+\; \mathcal{O}(1/k^2). \tag{A.21}$$

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